Antiquarks:

Isospin doublet for quarks: $q = \begin{pmatrix} u \\ d \end{pmatrix}$

General SU(2) transformation: $q \to q' = Uq$

$$\begin{pmatrix} u \\ d \end{pmatrix} \to \begin{pmatrix} u' \\ d' \end{pmatrix} = \overbrace{\begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix}}^{U} \begin{pmatrix} u \\ d \end{pmatrix}, \quad \overbrace{aa^* + bb^*}^{\det U} = 1$$

Charge conjugation $\hat{C}\psi = i\gamma^2\psi^* \Rightarrow$ complex conjugation should give us the transformation properties of the flavour part of the antiquark wavefunction

$$\begin{pmatrix} \bar{u}' \\ \bar{d}' \end{pmatrix} = U^* \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix} = \begin{pmatrix} a^* & b^* \\ -b & a \end{pmatrix} \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix} \qquad (1)$$

We want the antiquarks to transform the same way as quarks: $\bar{q} \to \bar{q}' = U\bar{q}$

We can write the antiquark doublet as

$$\bar{q} = \begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix} = S \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix} = S^{-1}\bar{q} \qquad \begin{pmatrix} \bar{u}' \\ \bar{d}' \end{pmatrix} = S^{-1}\bar{q}'$$

We can write (1) as $\quad S^{-1}\bar{q}' = U^* S^{-1}\bar{q} \quad | \times S$ on the left

$$\bar{q}' = S U^* S^{-1} \bar{q}$$

$$s u^* s^{-1} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a^* & b^* \\ -b & a \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix} = U$$

$$\Rightarrow \bar{q} \to \bar{q}' = U \bar{q}$$